EE 508
Lecture 16

Filter Transformations

Lowpass to Highpass
Lowpass to Band-reject

Filter Synthesis

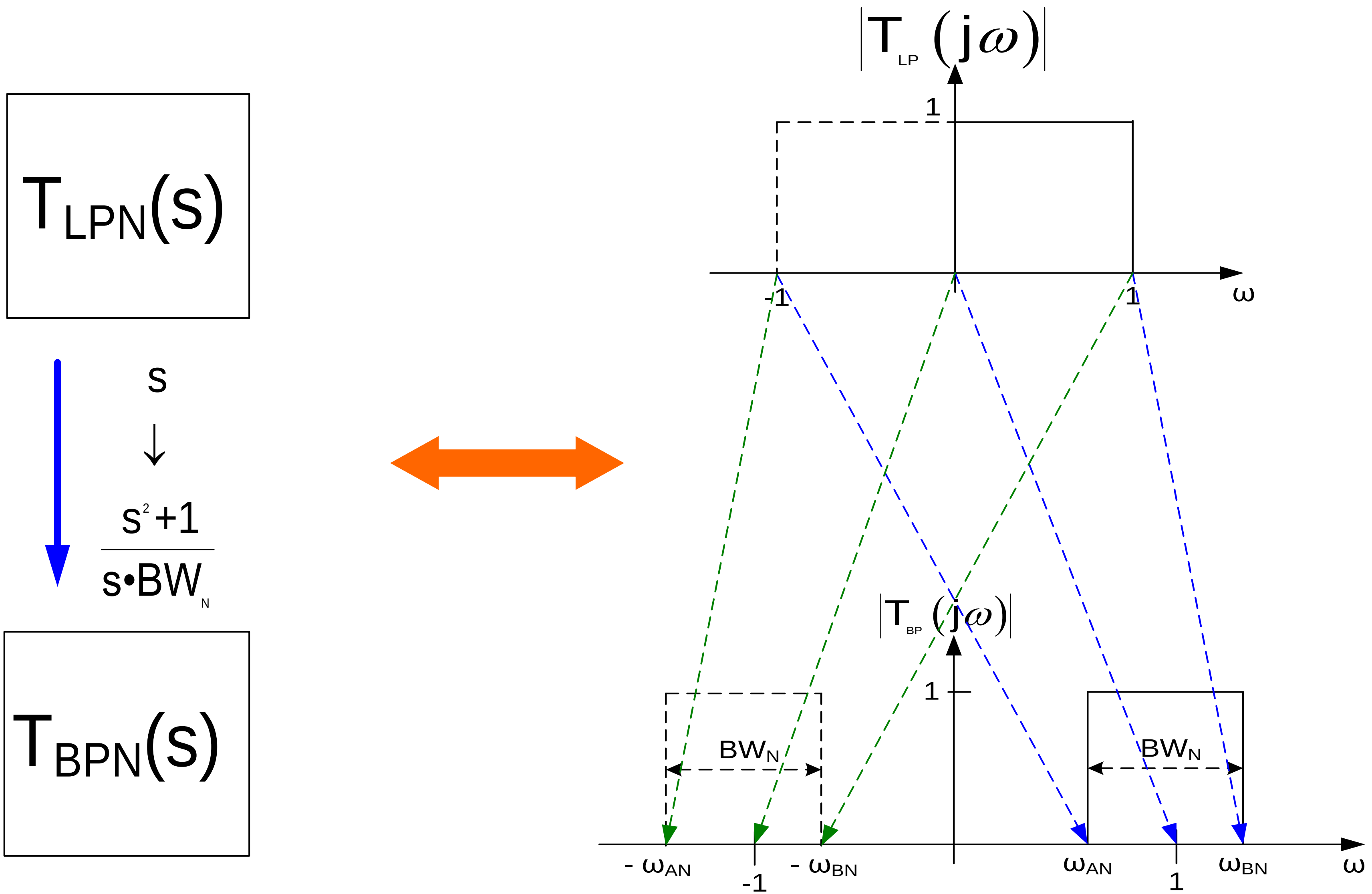
Standard LP to BP Transformation

$$s \to \frac{s^2+1}{s \bullet BW_N}$$

- Standard LP to BP transform is a variable mapping transform
- Maps jω axis to jω axis
- Maps LP poles to BP poles
- Preserves basic shape but warps frequency axis
- Doubles order
- Pole Q of resultant band-pass functions can be very large for narrow pass-band
- Sequencing of frequency scaling and transformation does not affect final function

Review from Last Time

Standard LP to BP Transformation



Review from Last Time

Standard LP to BP Transformation

Frequency and s-domain Mappings - Denormalized

(subscript variable in LP approximation for notational convenience)

$T_{LPN}(s_X)$ → $s_X \rightarrow \dfrac{s}{W_N}$ → $T_{LP}(s)$ → $s \rightarrow \dfrac{s^2+1}{s \bullet BW_N}$ → $T_{BP}(s)$

$T_{LPN}(s_X)$ → $s_X \rightarrow \dfrac{s_N^2+1}{s_N \bullet BW_N}$ → $T_{BPN}(s_N)$ → $s_N \rightarrow \dfrac{s}{W_N}$ → $T_{BP}(s)$

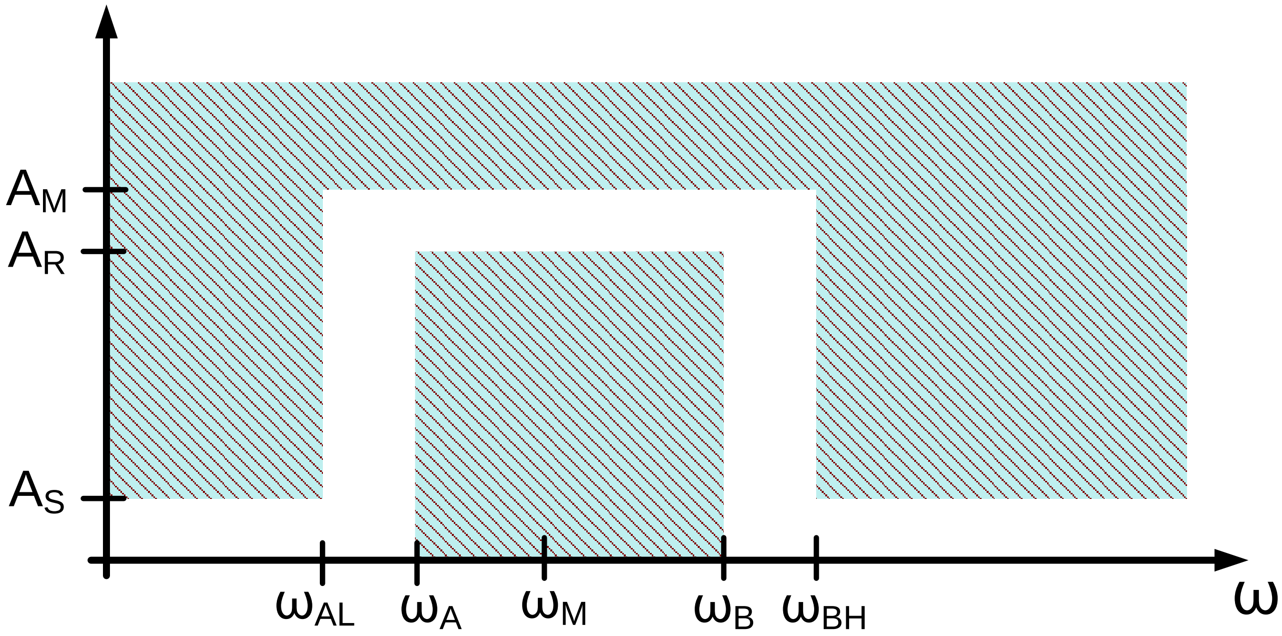
$T_{LPN}(s_X)$ → $s_X \rightarrow \dfrac{s^2+\omega_M^2}{s \bullet BW}$ → $T_{BP}(s)$

All three approaches give same approximation

Which is most practical to use? Often none of them !

Review from Last Time

Example 1: Obtain an approximation that meets the following specifications



$$BW=\omega_B-\omega_A$$

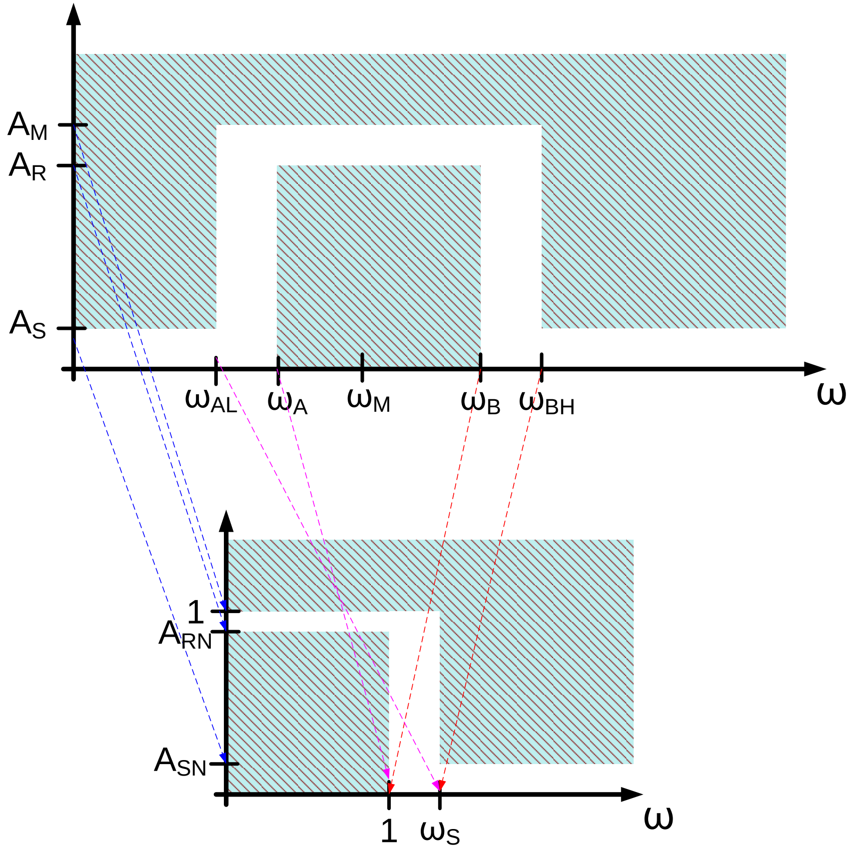
$$\omega_M=\sqrt{\omega_B\bullet\omega_A}$$

Assume that ω_{AL}, ω_{BH} and ω_M satsify

$$\frac{\omega_M^2-\omega_{AL}^2}{\omega_{AL}\bullet BW}=\frac{\omega_{BH}^2-\omega_M^2}{\omega_{BH}\bullet BW}$$

Review from Last Time

Example 1: Obtain an approximation that meets the following specifications



$$A_{RN}=\frac{A_R}{A_M}$$

$$A_{SN}=\frac{A_S}{A_M}$$

$$\frac{1}{\sqrt{1+\varepsilon^2}}=\frac{A_R}{A_M}$$

$$\varepsilon=\sqrt{\left(\frac{A_M}{A_R}\right)^2-1}$$

$$\omega_S=\frac{\omega_M^2-\omega_{AL}^2}{\omega_{AL}\bullet BW}$$

$$BW=\omega_B-\omega_A$$

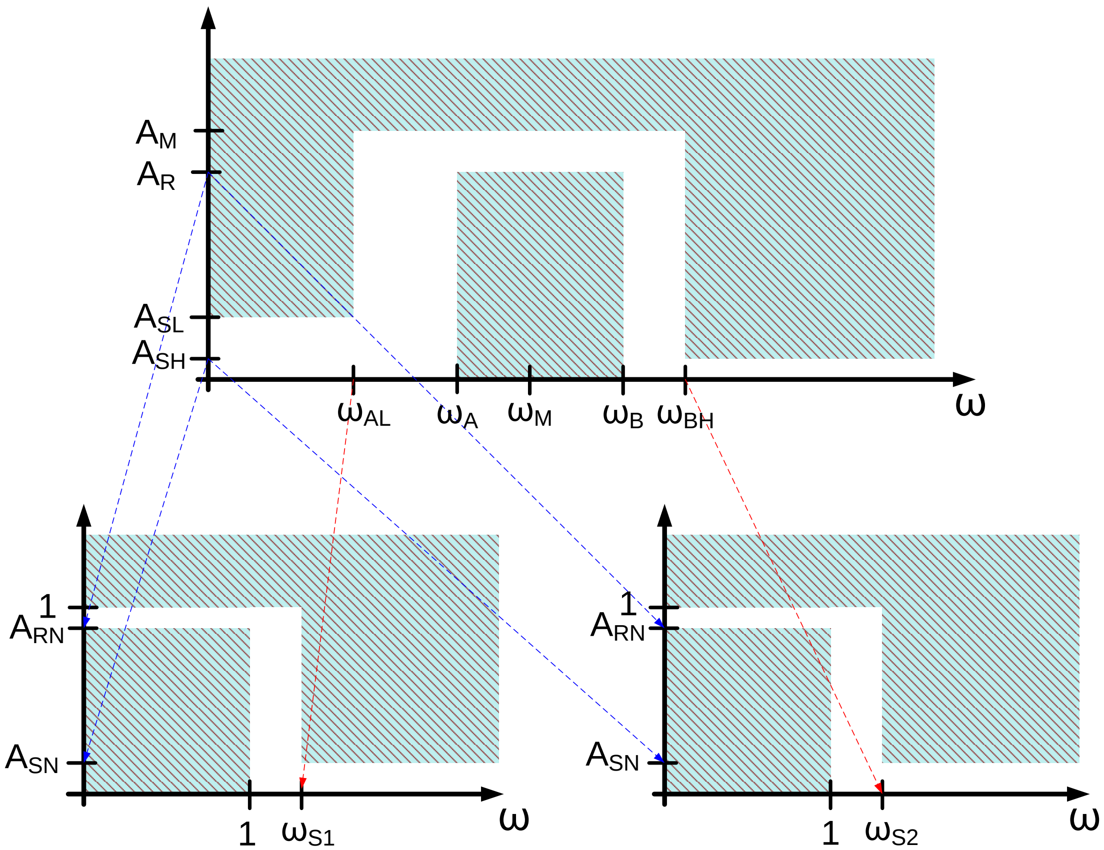
$$\omega_M=\sqrt{\omega_B\bullet\omega_A}$$

$$\frac{\omega_M^2-\omega_{AL}^2}{\omega_{AL}\bullet BW}=\frac{\omega_{BH}^2-\omega_M^2}{\omega_{BH}\bullet BW}$$

(actually ω_A and ω_{AL} that map to -1 and $-\omega_S$ respectively but show 1 and ω_S for convenience)

Review from Last Time

Example 2: Obtain an approximation that meets the following specifications



$$A_{RN}=\frac{A_R}{A_M}$$

$$\frac{1}{\sqrt{1+\varepsilon^2}}=\frac{A_R}{A_M}$$

$$A_{SN}=\min\left\{\frac{A_{SH}}{A_M},\frac{A_{SL}}{A_M}\right\}$$

$$\varepsilon=\sqrt{\left(\frac{A_M}{A_R}\right)^2-1}$$

$$\omega_{S1}=\frac{\omega_M^2-\omega_{AL}^2}{\omega_{AL}\bullet BW}$$

$$\omega_{S2}=\frac{\omega_{BH}^2-\omega_M^2}{\omega_{BH}\bullet BW}$$

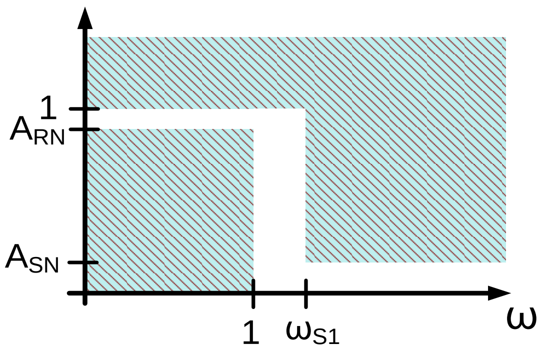
$$BW=\omega_B-\omega_A$$

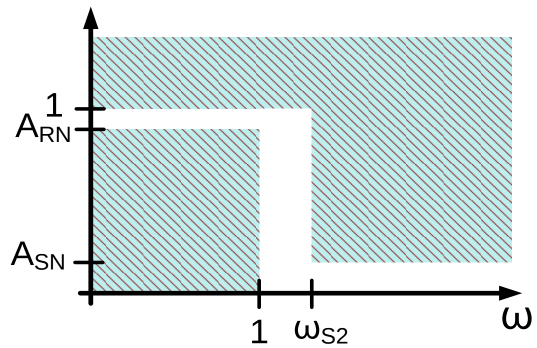
$$\omega_M=\sqrt{\omega_B\bullet\omega_A}$$

$$\omega_{SN}=\min\{\omega_{S1},\omega_{S2}\}$$

Review from Last Time

Example 2: Obtain an approximation that meets the following specifications





$$A_{RN}=\frac{A_R}{A_M}$$

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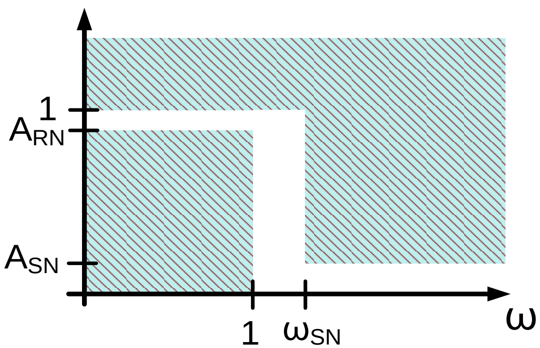


$$\omega_{SN}=\min\{\omega_{S1},\omega_{S2}\}$$

$$\varepsilon=\sqrt{\left(\frac{A_M}{A_R}\right)^2-1}$$

$$\omega_{S1}=\frac{\omega_M^2-\omega_{AL}^2}{\omega_{AL}\bullet BW}$$

$$\omega_{S2}=\frac{\omega_{BH}^2-\omega_M^2}{\omega_{BH}\bullet BW}$$

$$BW=\omega_B-\omega_A$$

$$\omega_M=\sqrt{\omega_B\bullet\omega_A}$$

$$\omega_{SN}=\min\{\omega_{S1},\omega_{S2}\}$$

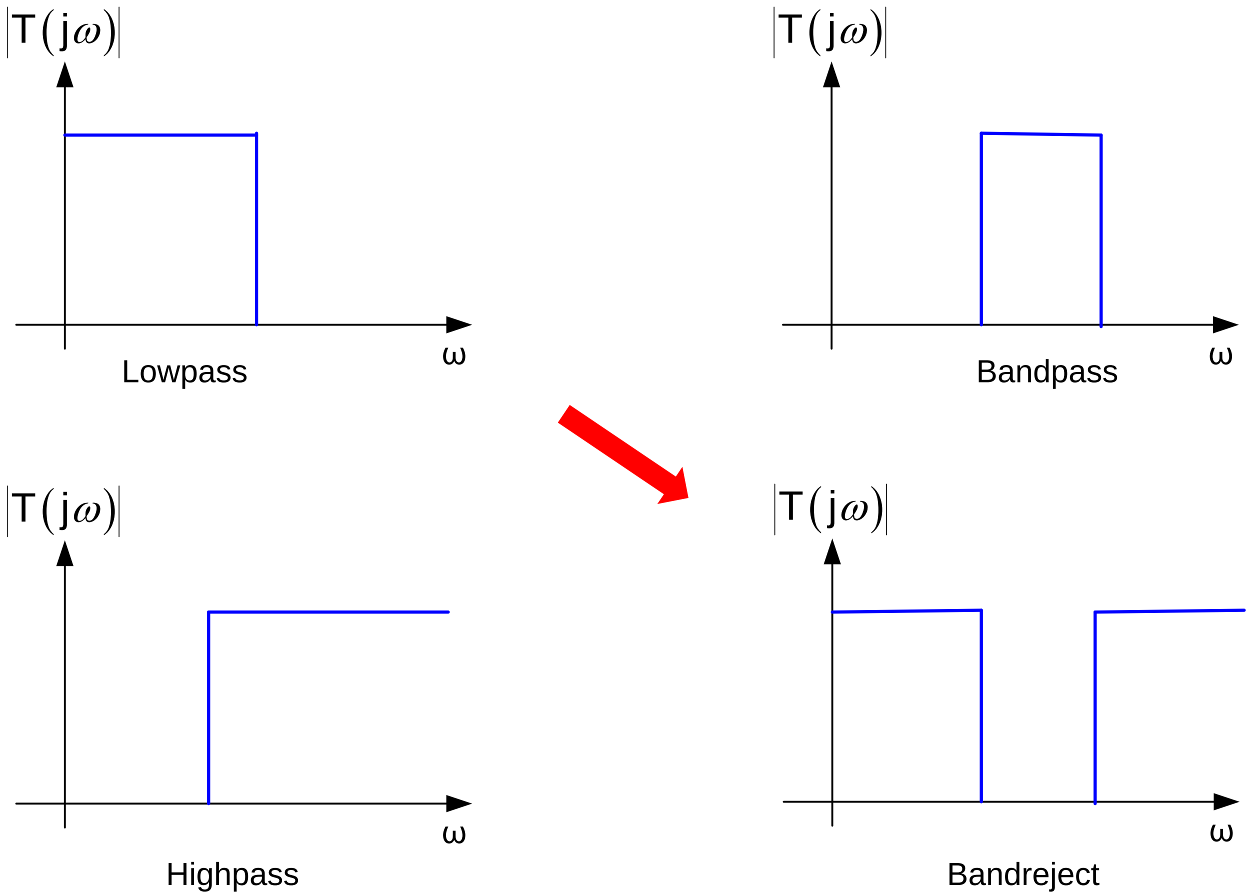
Filter Transformations

Lowpass to Bandpass **(LP to BP)**
Lowpass to Highpass **(LP to HP)**
→ Lowpass to Band-reject **(LP to BR)**

- Approach will be to take advantage of the results obtained for the standard LP approximations

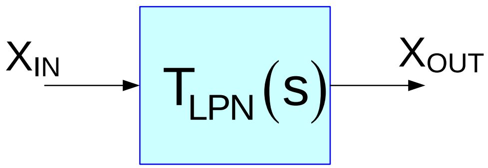
- Will focus on flat passband and zero-gain stop-band transformations

Flat Passband/Stopband Filters

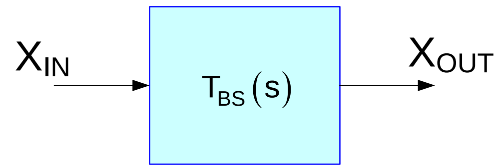


LP to BS Transformation

Strategy: As was done for the LP to BP approximations, will use a variable mapping strategy that maps the imaginary axis in the s-plane to the imaginary axis in the s-plane so the basic shape is preserved.



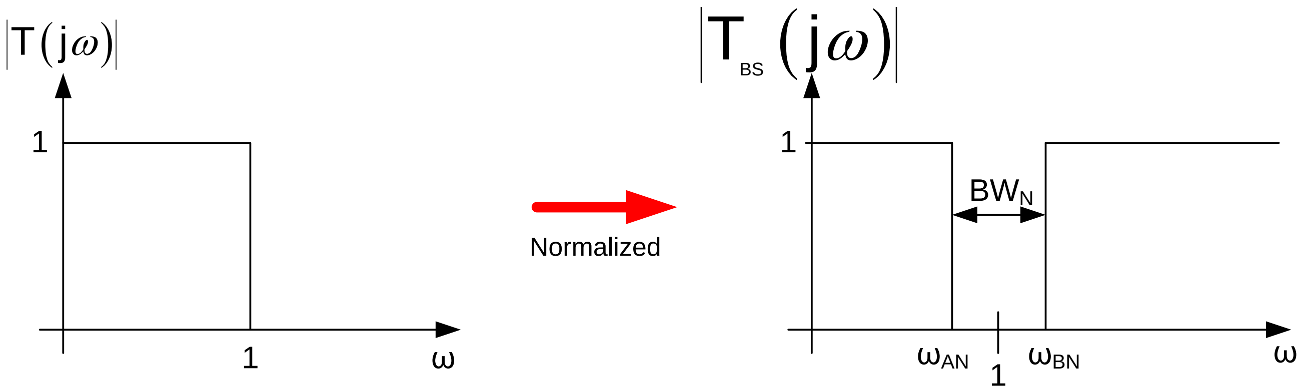
$$s \rightarrow f(s)$$



$$T_{BS}(s) = T_{LPN}(f(s))$$

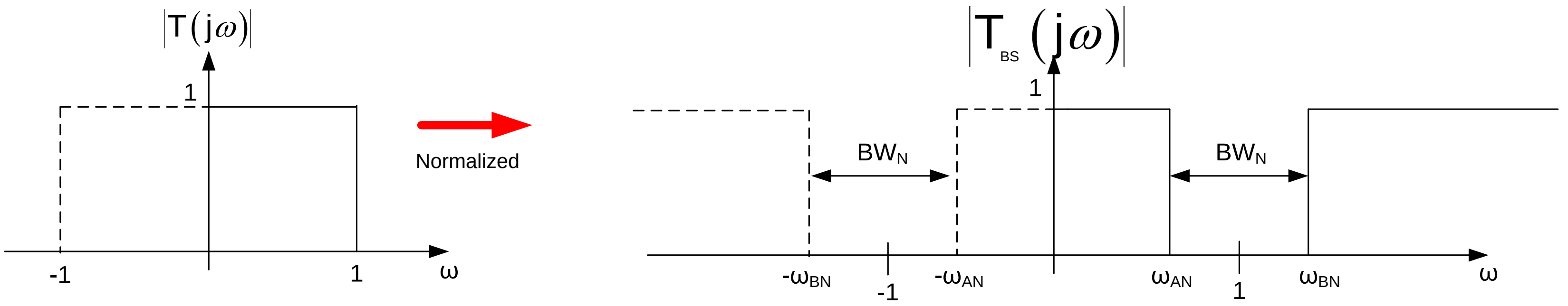
$$f(s) = \frac{\sum_{i=0}^{m_T} a_{Ti} s^i}{\sum_{i=0}^{n_T} b_{Ti} s^i}$$

LP to BS Transformation



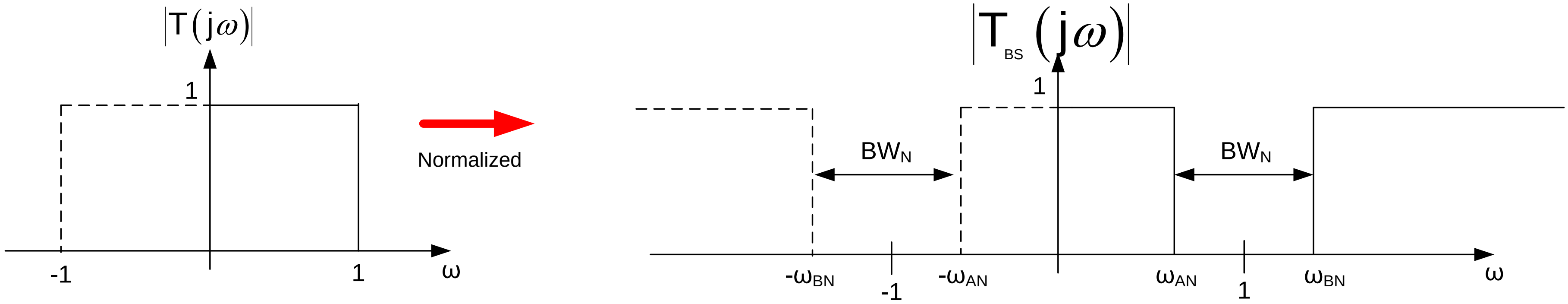
$$BW_N = \omega_{BN} - \omega_{AN}$$

$$\sqrt{\omega_{AN}\omega_{BN}} = 1$$



Standard LP to BS Transformation

Mapping Strategy:



Variable Mapping Strategy to Preserve Shape of LP function:

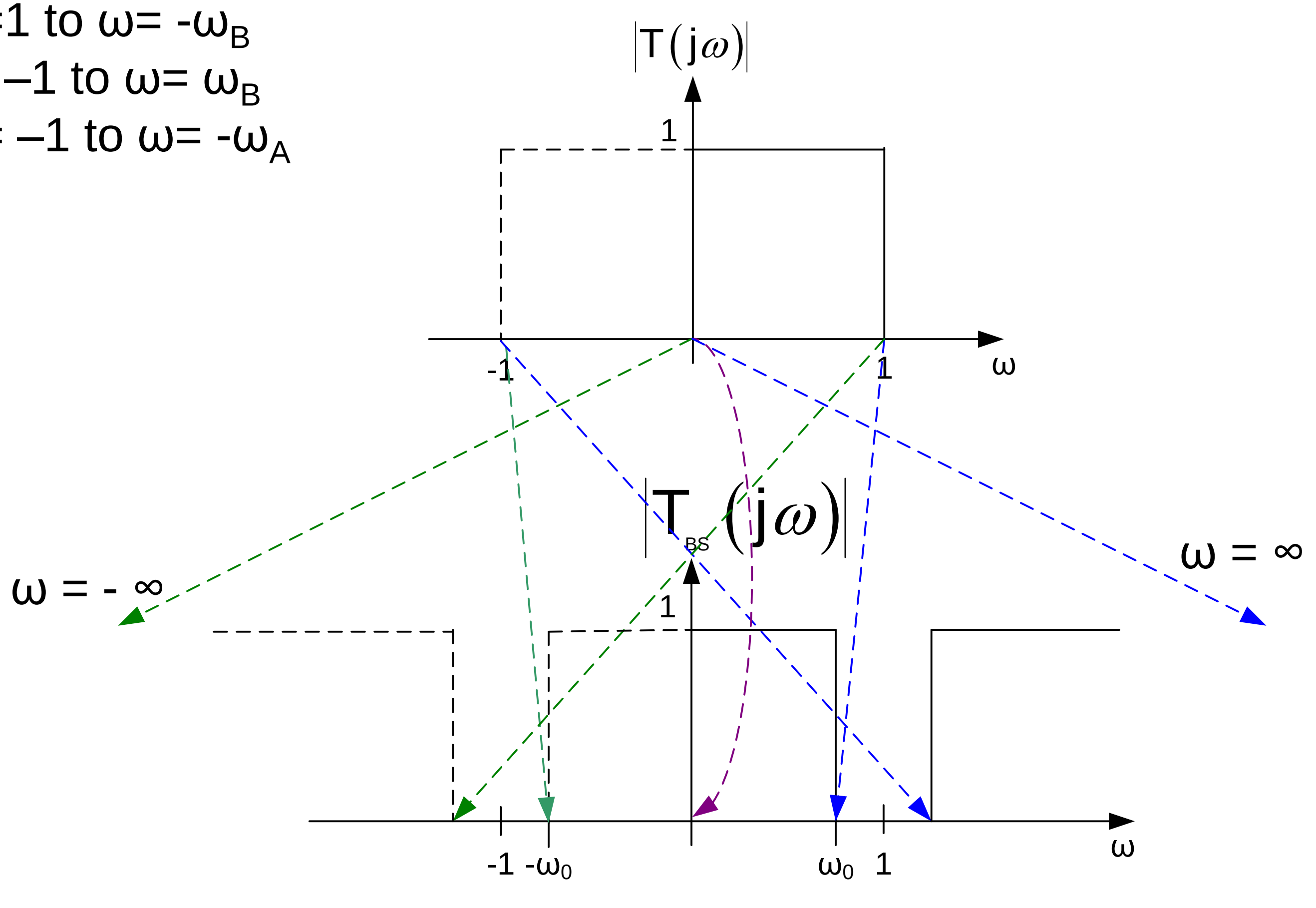
$F_N(s)$ should

map s=0 to s=± j∞
map s=0 to s= j0
map s=j1 to s=jω_A
map s=j1 to s=-jω_B
map s=-j1 to s=jω_B
map s=-j1 to s=-jω_A

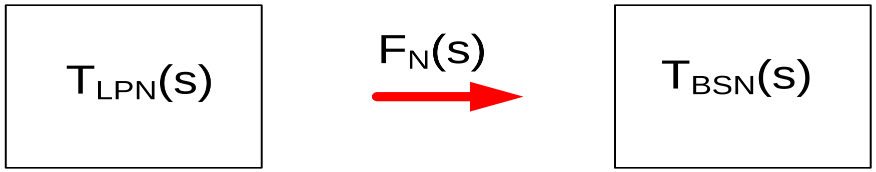
map ω=0 to ω = ±∞
map ω=0 to ω = 0
map ω=1 to ω = ω_A
map ω=1 to ω= -ω_B
map ω= –1 to ω= ω_B
map ω= –1 to ω= -ω_A

Standard LP to BS Transformation

map ω=0 to ω = ±∞
map ω=0 to ω = 0
map ω=1 to ω = ω_A
map ω=1 to ω= $-\omega_B$
map ω= –1 to ω= ω_B
map ω= –1 to ω= $-\omega_A$



Standard LP to BS Transformation



Mapping Strategy: consider variable mapping transform

$F_N(s)$ should

map s=0 to s=± j∞
map s=0 to s= j0
map s=j1 to s=jω_A
map s=j1 to s=-jω_B
map s=-j1 to s=jω_B
map s=-j1 to s=-jω_A

map ω=0 to ω = ±∞
map ω=0 to ω = 0
map ω=1 to ω = ω_A
map ω=1 to ω= -ω_B
map ω= –1 to ω= ω_B
map ω= –1 to ω= -ω_A

Consider variable mapping

$$T_{LPN}\left(F_N(s)\right)=T_{BSN}(s)\Big|_{s=\frac{s\bullet BW_N}{s^2+1}}$$

$$s \rightarrow \frac{s\bullet BW_N}{s^2+1}$$

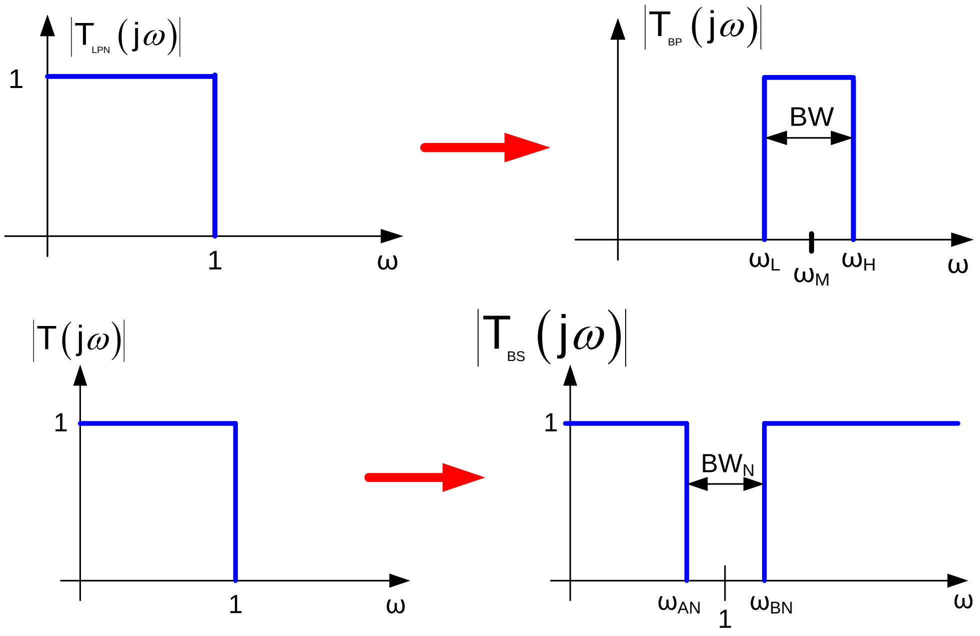
Comparison of Transforms

LP to BP

$$s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$

LP to BS

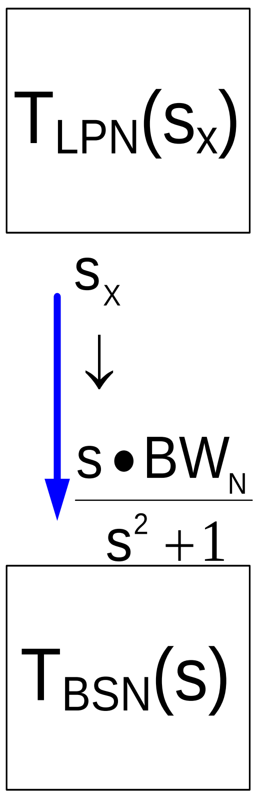
$$s \rightarrow \frac{s \bullet BW_N}{s^2+1}$$



Standard LP to BS Transformation

Frequency and s-domain Mappings

(subscript variable in LP approximation for notational convenience)



$$s_X \rightarrow \frac{s \bullet BW_N}{s^2+1}$$

$$\omega_X \rightarrow \frac{\omega \bullet BW_N}{1-\omega^2}$$

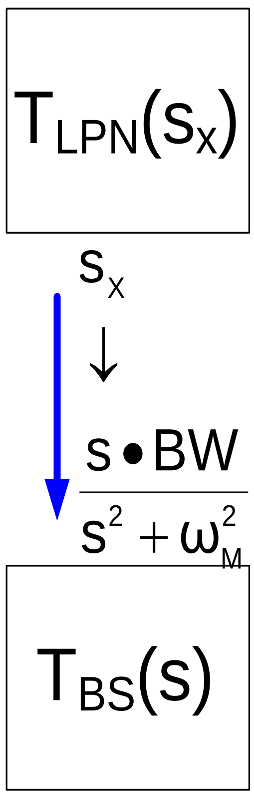
$$s \leftarrow \frac{1}{2}\frac{BW_N}{s_X} \pm \frac{1}{2}\sqrt{\left(\frac{BW_N}{s_X}\right)^2 - 4}$$

$$\omega \leftarrow \frac{-1}{2}\frac{BW_N}{\omega_X} \pm \frac{1}{2}\sqrt{\left(\frac{BW_N}{\omega_X}\right) + 4}$$

Standard LP to BS Transformation

Un-normalized Frequency and s-domain Mappings

(subscript variable in LP approximation for notational convenience)



$$s_X \rightarrow \frac{s \bullet BW}{s^2+\omega_M^2}$$

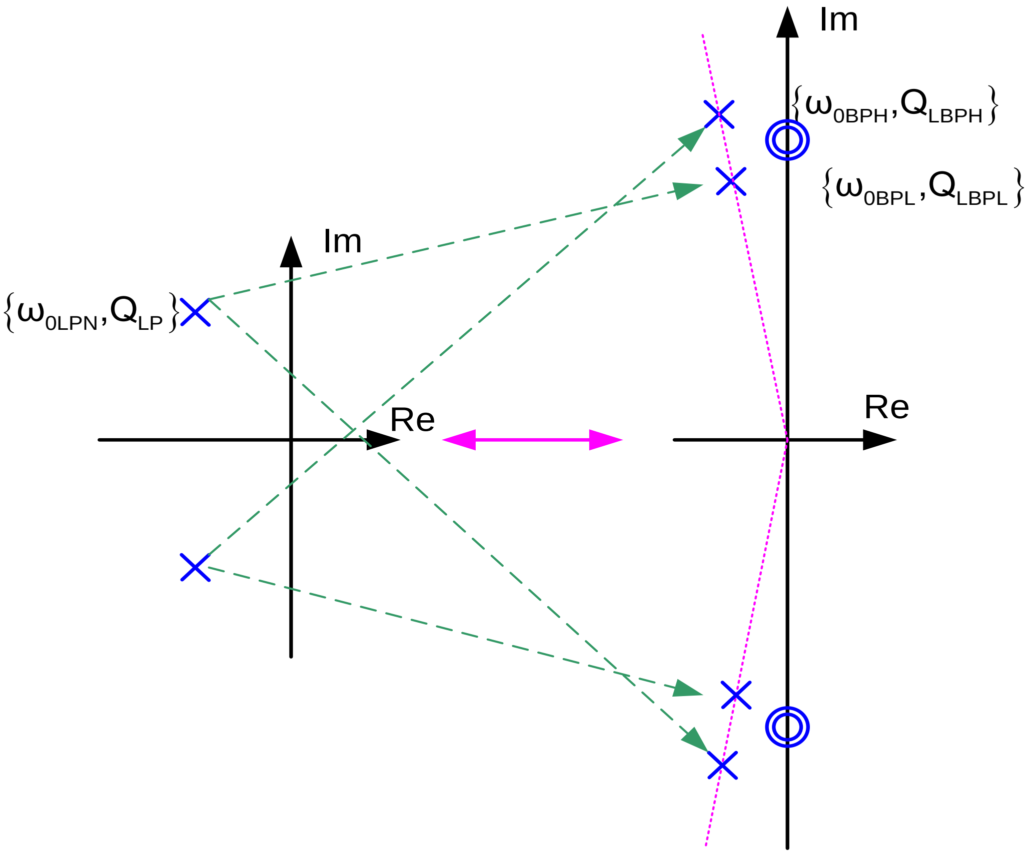
$$\omega_X \rightarrow \frac{\omega \bullet BW}{\omega_M^2-\omega^2}$$

$$s \leftarrow \frac{1}{2}\frac{BW}{s_X} \pm \frac{1}{2}\sqrt{\left(\frac{BW}{s_X}\right)^2 - 4\omega_M^2}$$

$$\omega \leftarrow \frac{-1}{2}\frac{BW}{\omega_X} \pm \frac{1}{2}\sqrt{\left(\frac{BW}{\omega_X}\right) + 4\omega_M^2}$$

Standard LP to BS Transformation

Pole Mappings

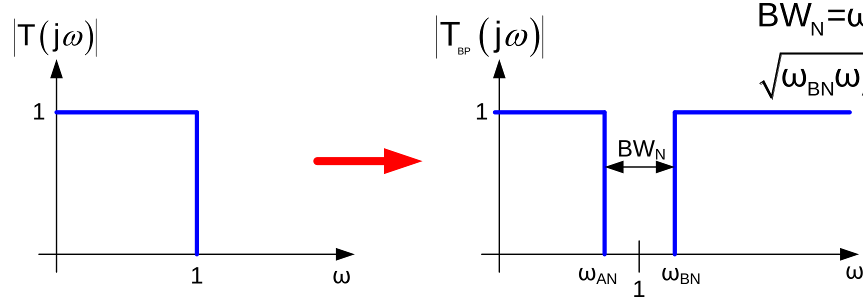


Can show that the upper hp pole maps to one upper hp pole and one lower hp pole as shown. Corresponding mapping of the lower hp pole is also shown

- Poles lie on a constant-Q line
- Zeros at ± j1 (normalized) or at $\pm j\omega_M$ (un-normalized) of multiplicity n

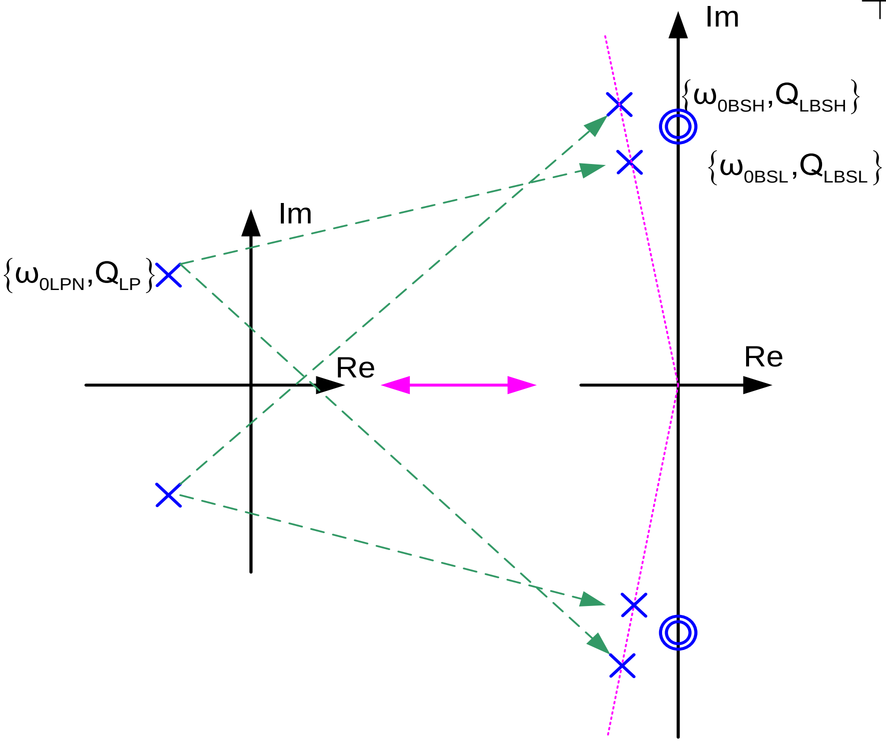
LP to BS Transformation

Pole Q of BS Approximations



$BW_N = \omega_{BN} - \omega_{AN}$

$\sqrt{\omega_{BN}\omega_{AN}} = 1$



Define: $\gamma = \left(\dfrac{BW}{\omega_M \omega_{0LPN}}\right)$ $\qquad BW = \omega_B - \omega_A$ $\qquad \sqrt{\omega_B \omega_A} = \omega_M$

It can be shown that

$$Q_{BSL} = Q_{BSH} = \frac{Q_{LP}}{\sqrt{2}}\sqrt{1 + \frac{4}{\gamma^2} + \sqrt{\left(1 + \frac{4}{\gamma^2}\right)^2 - \frac{4}{\gamma^2 Q_{LP}^2}}}$$

For γ small, $\qquad Q_{BS} \cong \dfrac{2Q_{LP}}{\gamma}$

It can be shown that

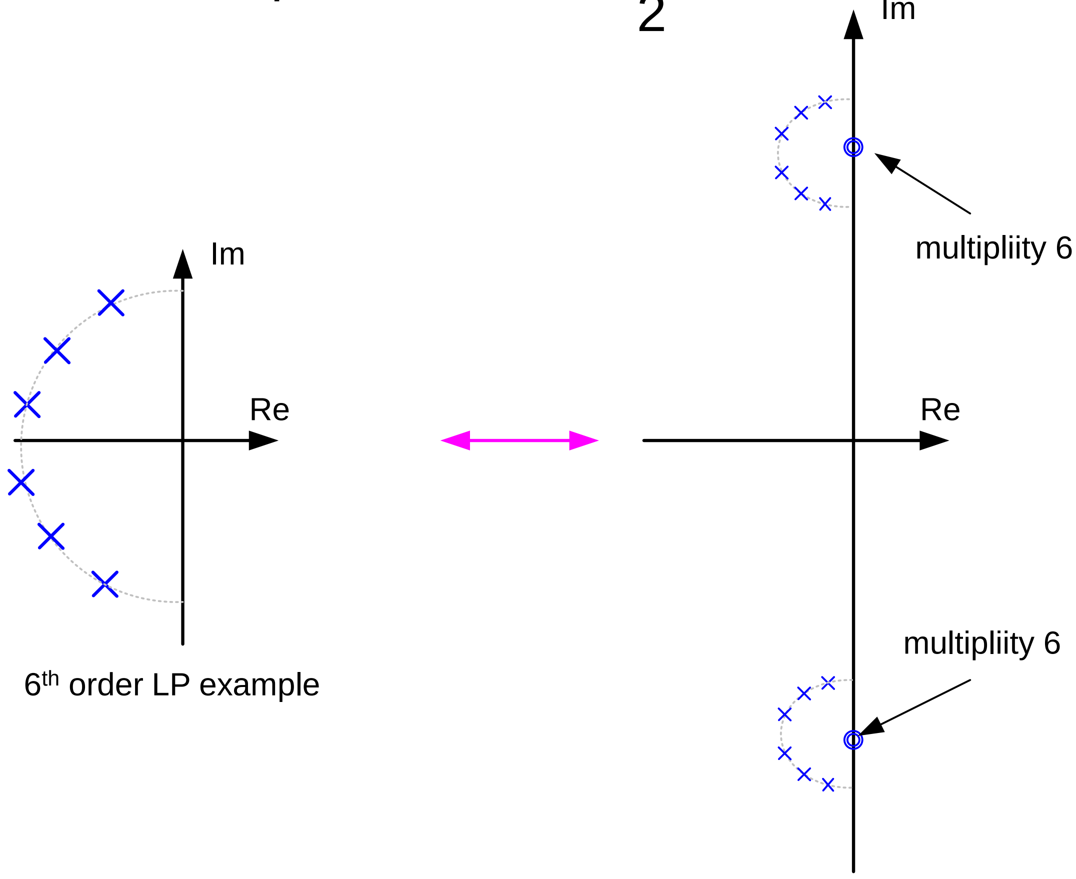
$$\omega_{0BS} = \frac{\omega_M}{2}\left[\gamma\frac{Q_{BS}}{Q_{LP}} \pm \sqrt{\left(\gamma\frac{Q_{BS}}{Q_{LP}}\right)^2 - 4}\right]$$

Note for γ **small, Q_{BS} can get very large**

Standard LP to BS Transformation

Pole Mappings

$$p \leftarrow \frac{\frac{BW_N}{p_X} \pm \sqrt{\left(\frac{BW_N}{p_X}\right)^2 - 4}}{2}$$



Note doubling of poles, addition of zeros, and likely Q enhancement

Standard LP to BS Transformation

$$s_X \rightarrow \frac{s \bullet BW}{s^2 + \omega_M^2}$$

- Standard LP to BS transformation is a variable mapping transform
- Maps jω axis to jω axis in the s-plane
- Preserves basic shape of an approximation but warps frequency axis
- Order of BS approximation is double that of the LP Approximation
- Pole Q and ω_0 expressions are identical to those of the LP to BP transformation
- Pole Q of BS approximation can get very large for narrow BW
- Other variable transforms exist but the standard is by far the most popular

Filter Transformations

Lowpass to Bandpass **(LP to BP)**

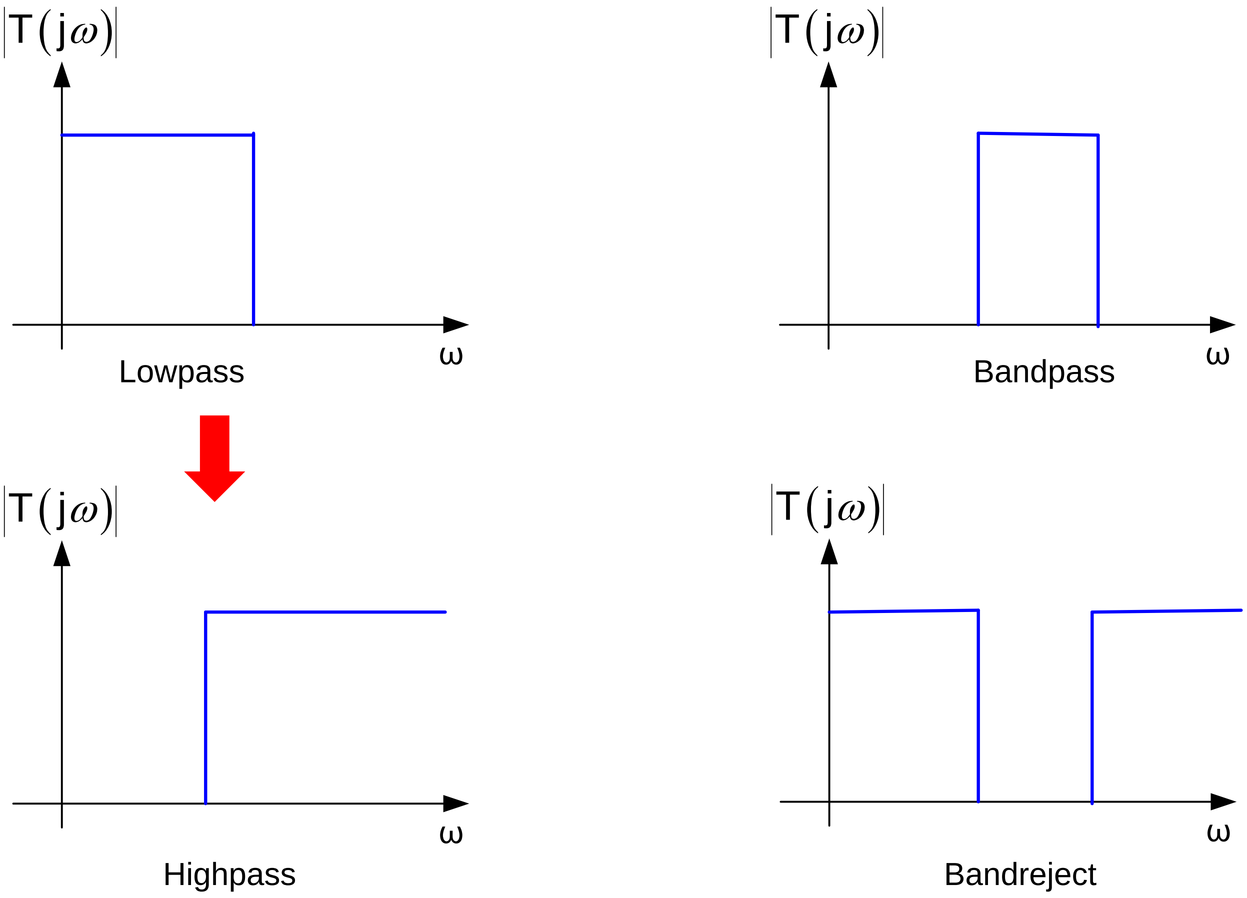
→ Lowpass to Highpass **(LP to HP)**

Lowpass to Band-reject **(LP to BR)**

- Approach will also be to take advantage of the results obtained for the standard LP approximations

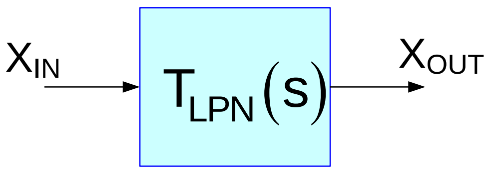
- Will focus on flat passband and zero-gain stop-band transformations

Flat Passband/Stopband Filters

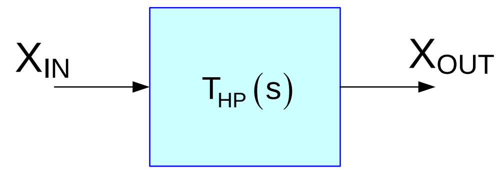


LP to HP Transformation

Strategy: As was done for the LP to BP approximations, will use a variable mapping strategy that maps the imaginary axis in the s-plane to the imaginary axis in the s-plane so the basic shape is preserved.



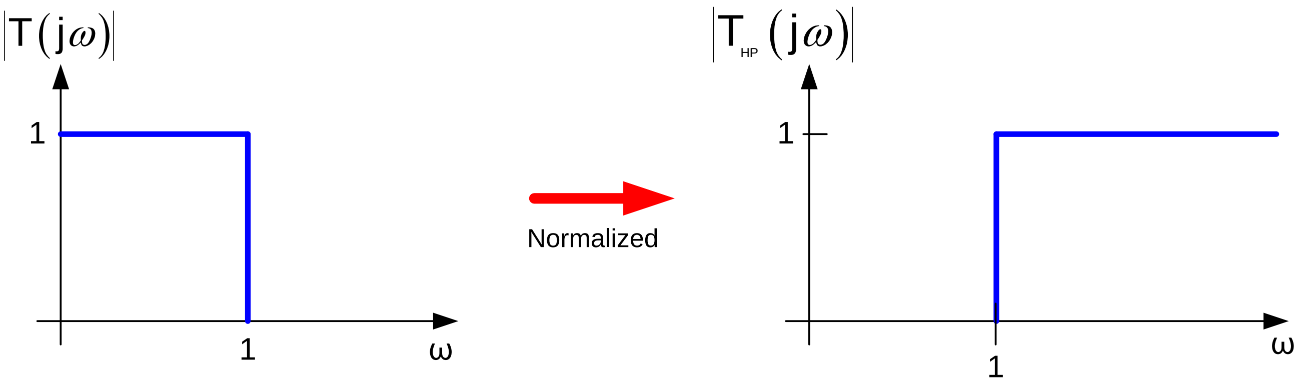
$$s \rightarrow f(s)$$

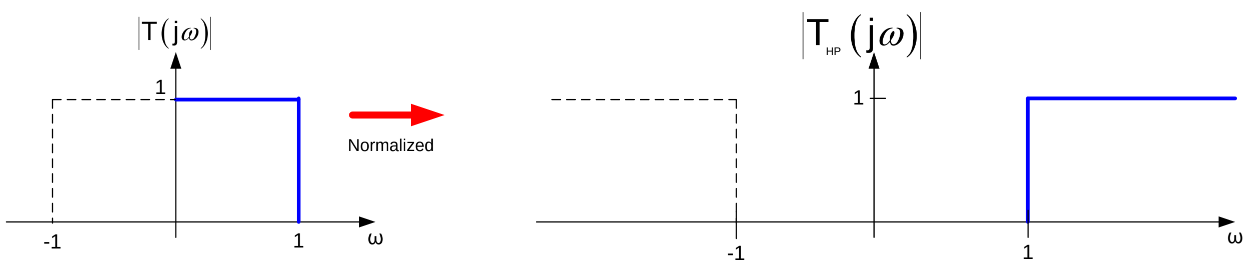


$$T_{HP}(s) = T_{LPN}(f(s))$$

$$f(s) = \frac{\sum_{i=0}^{m_T} a_{Ti} s^i}{\sum_{i=0}^{n_T} b_{Ti} s^i}$$

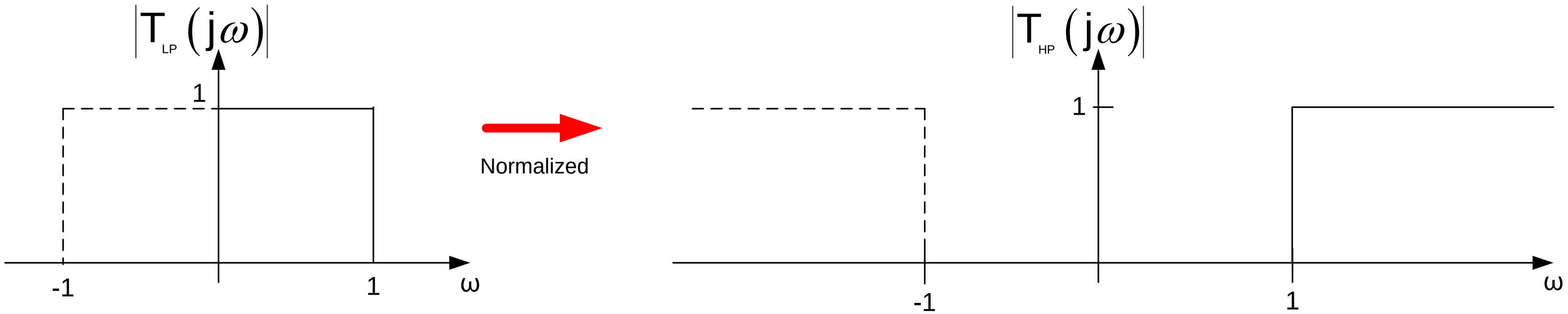
LP to HP Transformation





Standard LP to HP Transformation

Mapping Strategy:



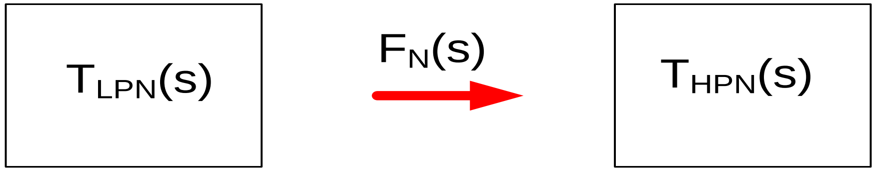
Variable Mapping Strategy to Preserve Shape of LP function:

$F_N(s)$ should

map s=0 to s=± j∞
map s=j1 to s=-j1
map s= –j1 to s=j1

map ω=0 to ω=∞
map ω=1 to ω=-1
map ω= –1 to ω=1

Standard LP to HP Transformation



Mapping Strategy: consider variable mapping transform

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map s=j1 to s=-j1
map s= –j1 to s=j1

map ω=0 to ω=∞
map ω=1 to ω=-1
map ω= –1 to ω=1

Consider variable mapping

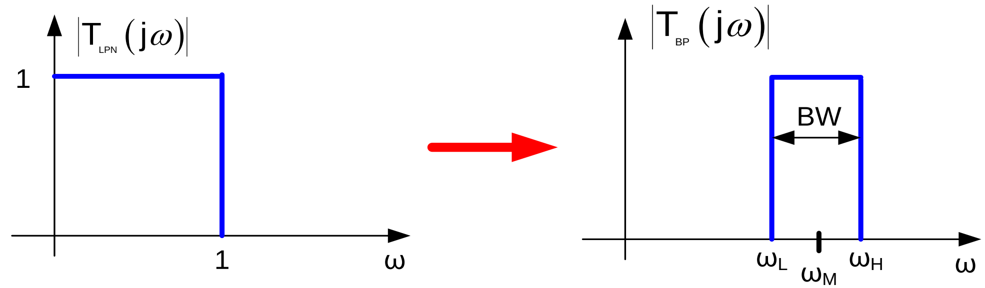
$$T_{LPN}(F(s)) = T_{LPN}(s)\Big|_{s=\frac{1}{s}}$$

$$s \rightarrow \frac{1}{s}$$

Comparison of Transforms

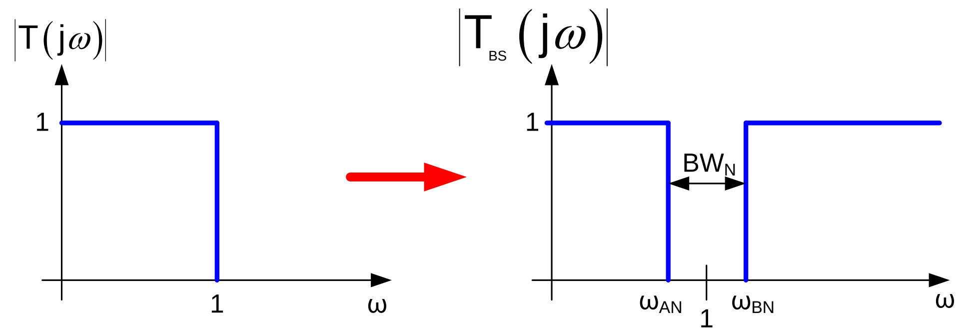
LP to BP

$$s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$



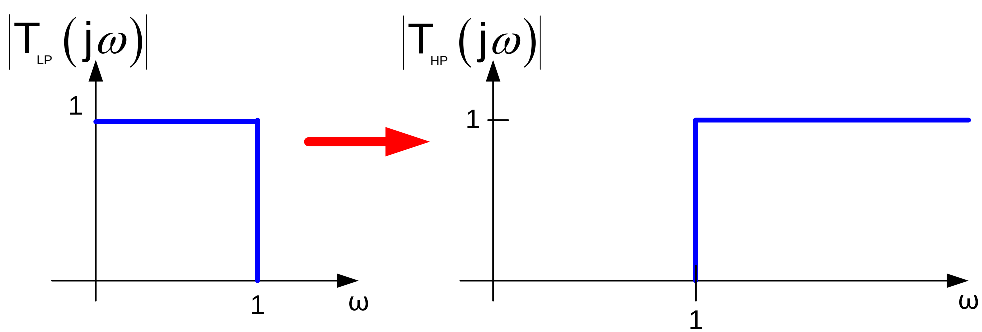
LP to BS

$$s \rightarrow \frac{s \bullet BW_N}{s^2+1}$$



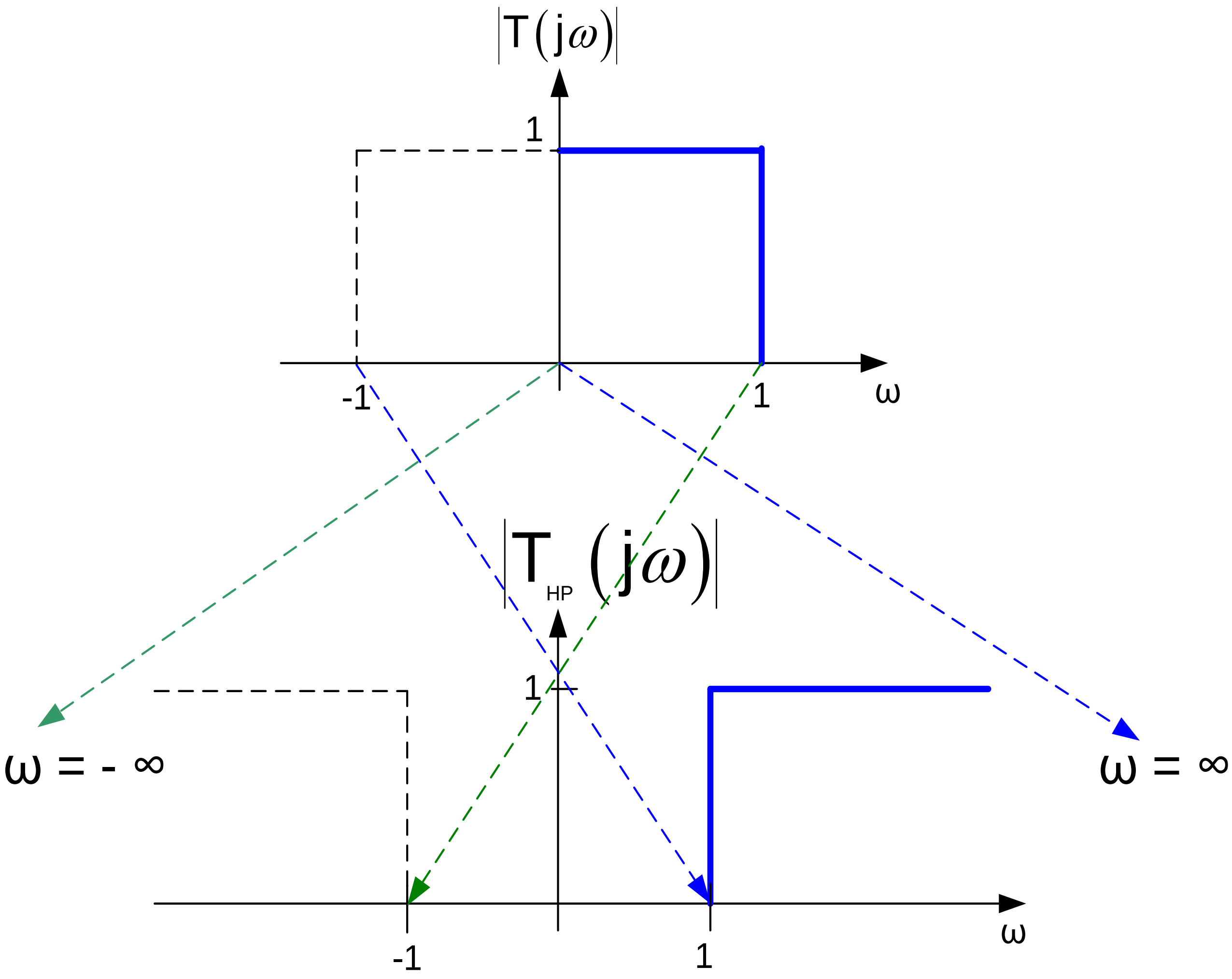
LP to HP

$$s \rightarrow \frac{1}{s}$$



LP to HP Transformation

(Normalized Transform)



Standard LP to HP Transformation

Frequency and s-domain Mappings

(subscript variable in LP approximation for notational convenience)

$T_{LPN}(s_X)$

$s_X \downarrow \dfrac{1}{s}$

$T_{HPN}(s)$

$$s_X \rightarrow \frac{1}{s}$$

$$\omega_X \rightarrow \frac{-1}{\omega}$$

$$s \leftarrow \frac{1}{s_X}$$

$$\omega \leftarrow \frac{-1}{\omega_X}$$

Standard LP to HP Transformation

Pole Mappings

Claim: With a variable mapping transform, the variable mapping naturally defines the mapping of the poles of the transformed function

$$T_{LPN}(s_X)$$

$$s_X \downarrow \frac{1}{s}$$

$$T_{HPN}(s)$$

$$p_X \to \frac{1}{p}$$

$$p \leftarrow \frac{1}{p_X}$$

Standard LP to HP Transformation

Pole Mappings

$T_{LPN}(s_X)$

$s_X \downarrow \frac{1}{s}$

$T_{HPN}(s)$

$$p \leftarrow \frac{1}{p_X}$$

If $p_X = \alpha + j\beta$ and $p_X = \alpha - j\beta$

$$p = \frac{1}{\alpha + j\beta} = \frac{\alpha - j\beta}{\alpha^2 + \beta^2}$$

$$p = \frac{1}{\alpha + j\beta} = \frac{\alpha + j\beta}{\alpha^2 + \beta^2}$$

Standard LP to HP Transformation

Pole Mappings

$T_{LPN}(s_X)$

$s_X \rightarrow \frac{1}{s}$

$T_{HPN}(s)$

$$p \leftarrow \frac{1}{p_X}$$

If $p_X = \alpha + j\beta$ and $p_X = \alpha - j\beta$

$$p = \frac{1}{\alpha + j\beta} = \frac{\alpha - j\beta}{\alpha^2 + \beta^2} \qquad p = \frac{1}{\alpha + j\beta} = \frac{\alpha + j\beta}{\alpha^2 + \beta^2}$$

Highpass poles are scaled in magnitude but make identical angles with imaginary axis

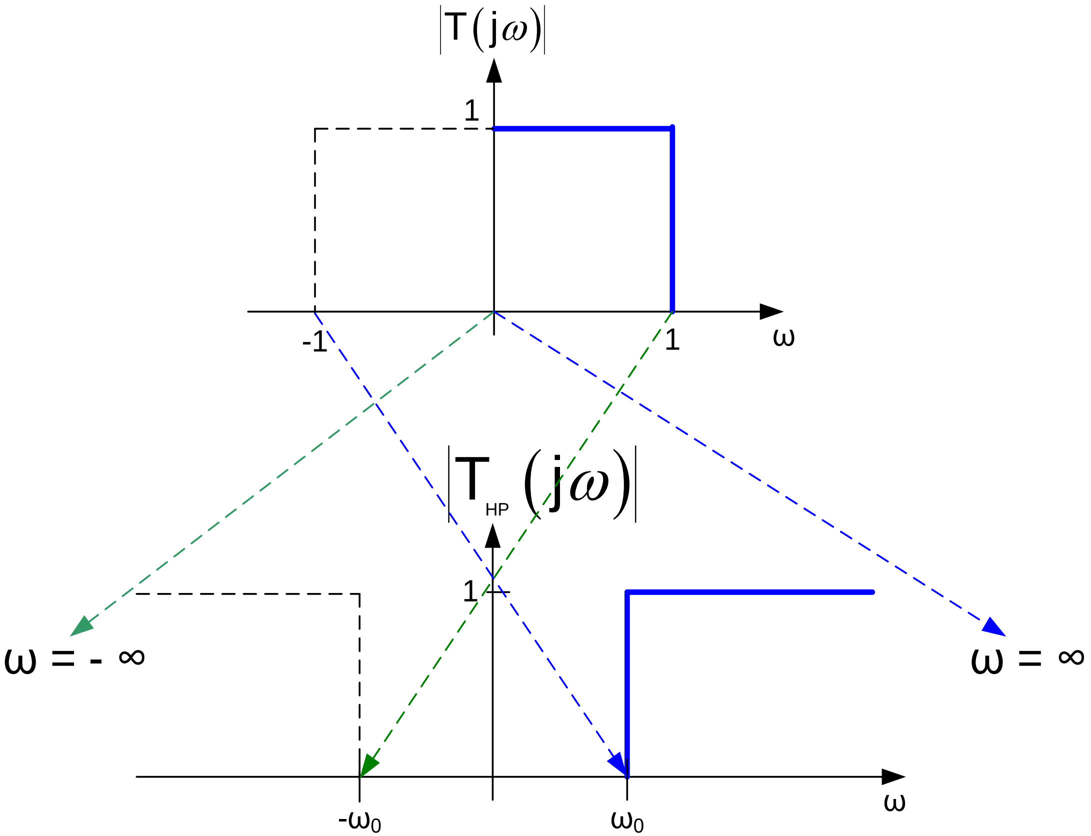
HP pole Q is same as LP pole Q

Order is preserved

Standard LP to HP Transformation

(Un-normalized variable mapping transform)

$$s \rightarrow \frac{\omega_0}{s}$$



Filter Design Process

Establish Specifications

- possibly $T_D(s)$ or $H_D(z)$
- magnitude and phase characteristics or restrictions
- time domain requirements

Approximation

- obtain acceptable transfer functions $T_A(s)$ or $H_A(z)$
- possibly acceptable realizable time-domain responses

Synthesis

- build circuit or implement algorithm that has response close to $T_A(s)$ or $H_A(z)$
- actually realize $T_R(s)$ or $H_R(z)$

Filter

Filter Design/Synthesis Considerations

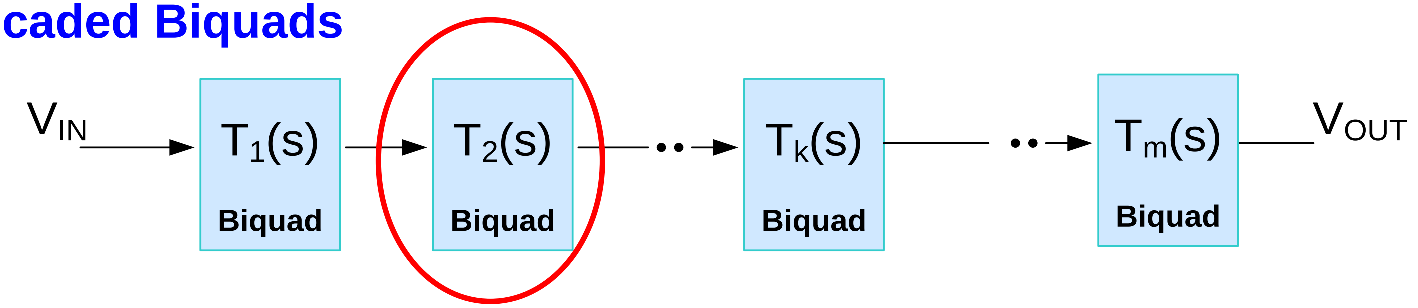
There are many different filter architectures that can realize a given transfer function

Considerable effort has been focused over the years on "inventing" these architectures and on determining which is best suited for a given application

Filter Design/Synthesis Considerations

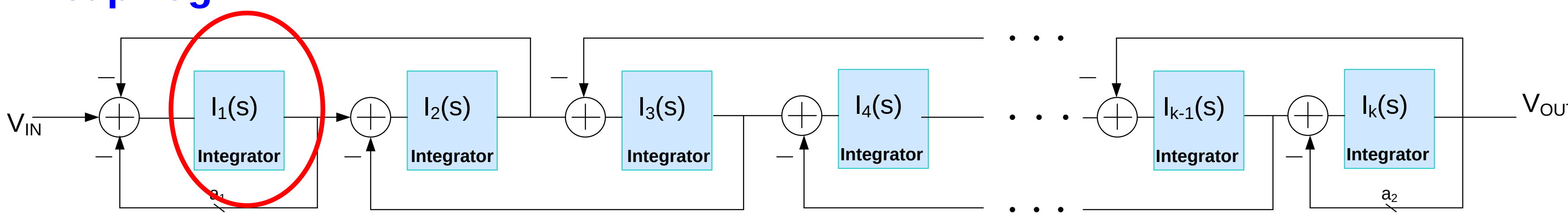
Most even-ordered designs today use one of the following three basic architectures

Cascaded Biquads

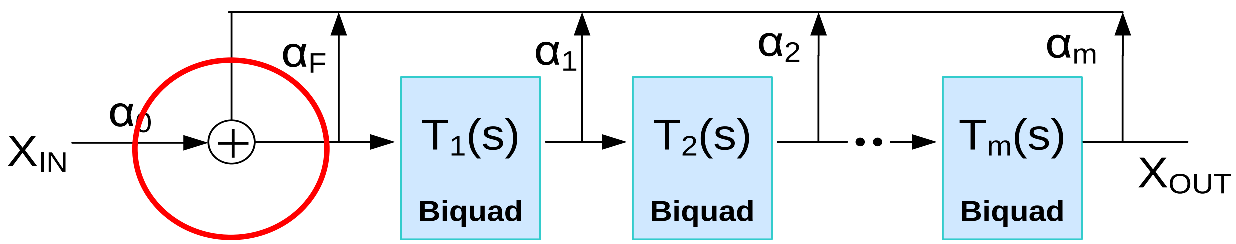


$$T(s) = T_1 T_2 \bullet\bullet T_m$$

Leapfrog



Multiple-loop Feedback (less popular)

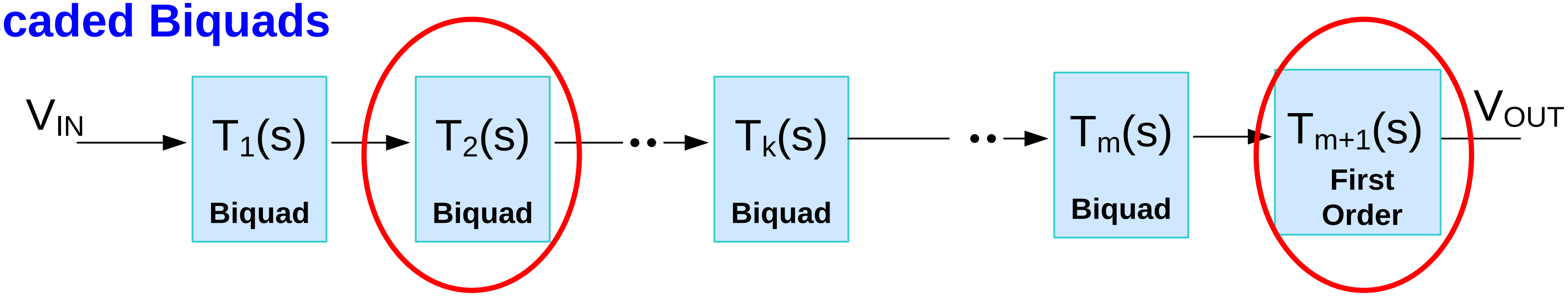


What's unique in all of these approaches?

Filter Design/Synthesis Considerations

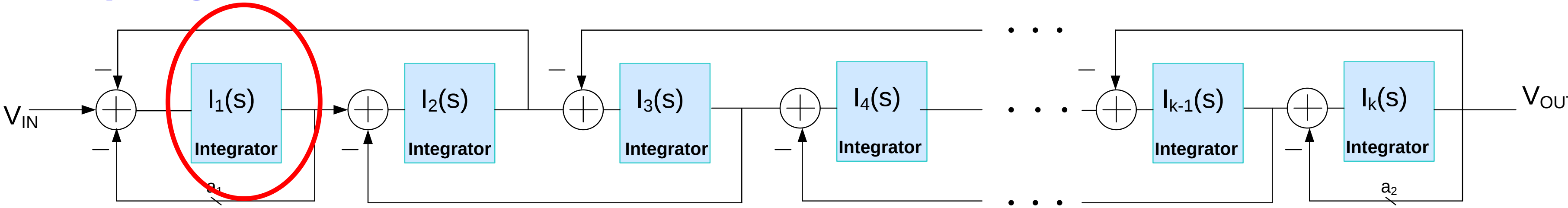
Most odd-ordered designs today use one of the following three basic architectures

Cascaded Biquads

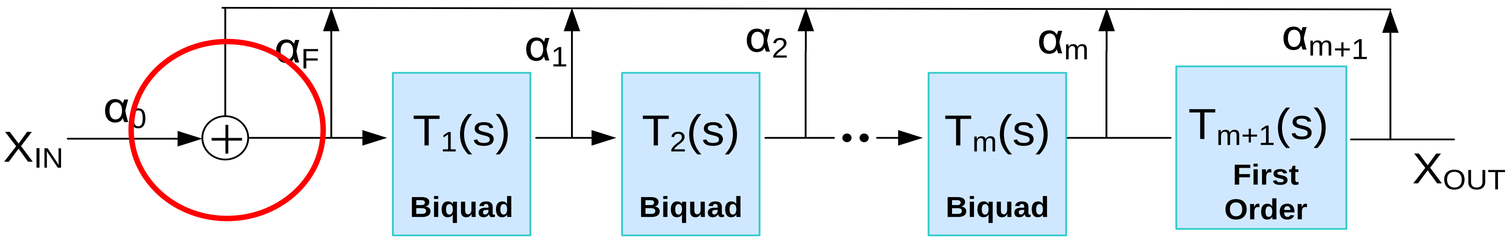


$$T(s) = T_1 T_2 \bullet\bullet T_m$$

Leapfrog



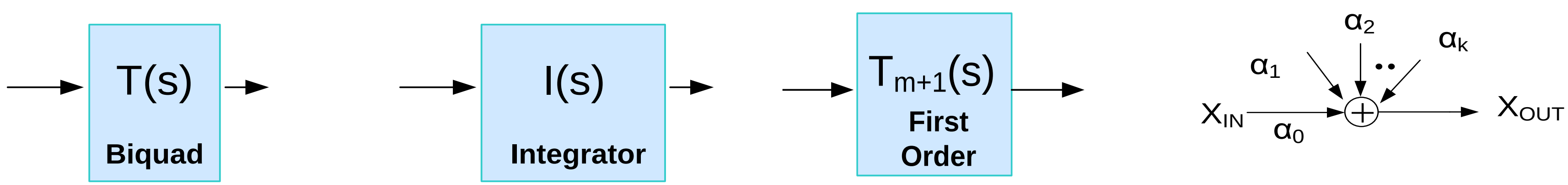
Multiple-loop Feedback (less popular)



What's unique in all of these approaches?

Filter Design/Synthesis Considerations

What's unique in all of these approaches?

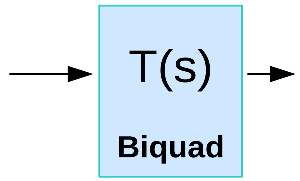


$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0} \qquad I(s) = \frac{I_0}{s} \qquad T(s) = \frac{a_1 s + a_0}{s + b_0} \qquad X_{OUT} = \sum_{i=0}^{k} \alpha_i$$

- Most effort on synthesis can focus on synthesizing these four blocks
  - (the summing function is often incorporated into the Biquad or Integrator)
  - (the first-order block is much less challenging to design than the biquad)
- Some issues associated with their interconnections
- And, in many integrated structures, the biquads are made with integrators
  - (thus, much filter design work simply focuses on the design of integrators)

Biquads

How many biquad filter functions are there?



$$T(s) = \frac{a_2s^2+a_1s+a_0}{s^2+b_1s+b_0}$$

$$T(s) = \frac{a_2s^2+a_1s+a_0}{s^2+b_1s+b_0} \qquad a_0 \neq 0,\ a_1 \neq 0,\ a_2 \neq 0$$

$$T(s) = \frac{a_0}{s^2+b_1s+b_0} \qquad a_0 \neq 0$$

$$T(s) = \frac{a_2s^2+a_0}{s^2+b_1s+b_0} \qquad a_0 \neq 0,\ a_2 \neq 0$$

$$T(s) = \frac{a_1s}{s^2+b_1s+b_0} \qquad a_1 \neq 0$$

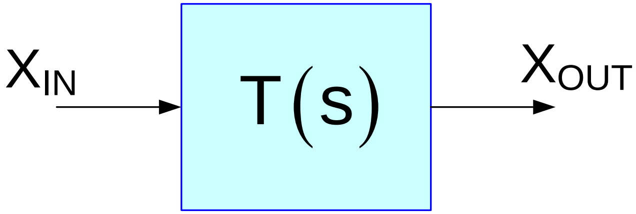
$$T(s) = \frac{a_1s+a_0}{s^2+b_1s+b_0} \qquad a_0 \neq 0,\ a_1 \neq 0$$

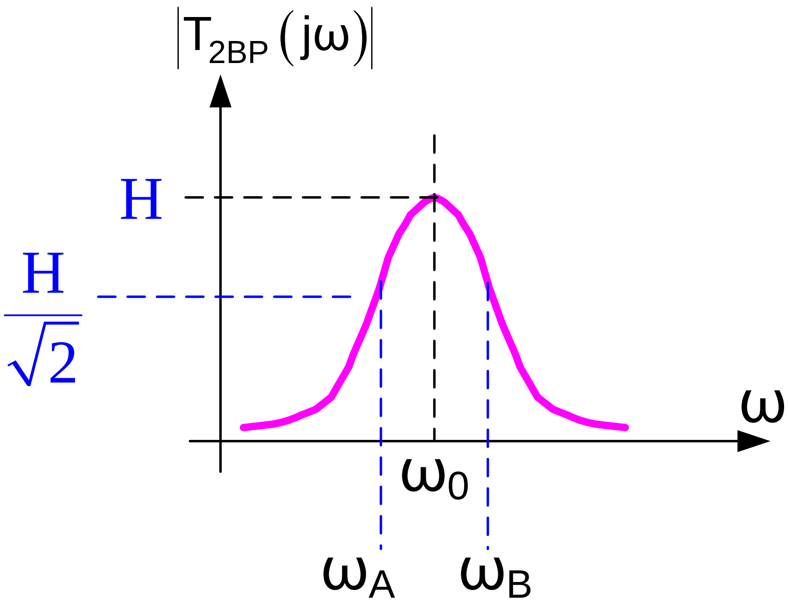
$$T(s) = \frac{a_2s^2}{s^2+b_1s+b_0} \qquad a_2 \neq 0$$

$$T(s) = \frac{a_2s^2+a_1s}{s^2+b_1s+b_0} \qquad a_2 \neq 0,\ a_1 \neq 0$$

Filter Design/Synthesis Considerations

Review: Second-order bandpass transfer function





$$\left|T_{2BP}(s)\right| = H \frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

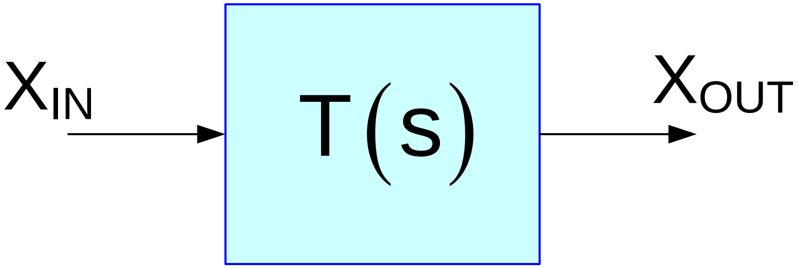
$$BW = \omega_B - \omega_A = \frac{\omega_0}{Q}$$

$$\omega_{PEAK} = \omega_0$$

Filter Design/Synthesis Considerations

There are many different filter architectures that can realize a given transfer function

Will first consider second-order Bandpass filter structures



$$\left|T(s)\right| = H\frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

$$BW = \omega_B - \omega_A = \frac{\omega_0}{Q}$$

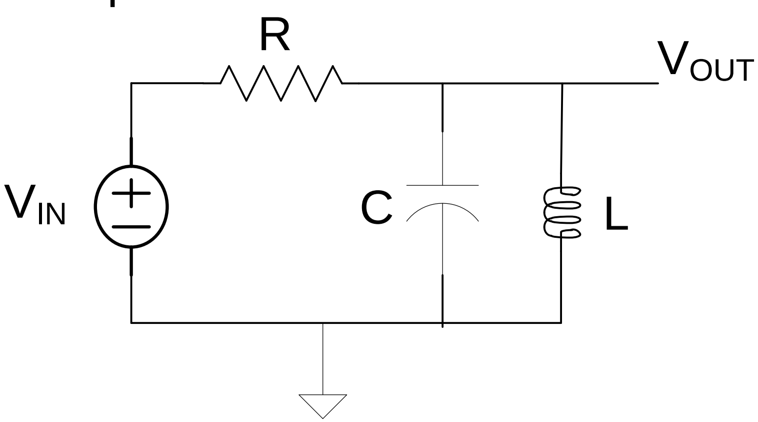
$$\omega_{PEAK} = \omega_0$$

Filter Design/Synthesis Considerations

There are many different filter architectures that can realize a given transfer function

Will first consider second-order Bandpass filter structures

Example 1:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{RC}\frac{s}{s^2 + s\left(\frac{1}{RC}\right) + \frac{1}{LC}}$$

Second-order Bandpass Filter

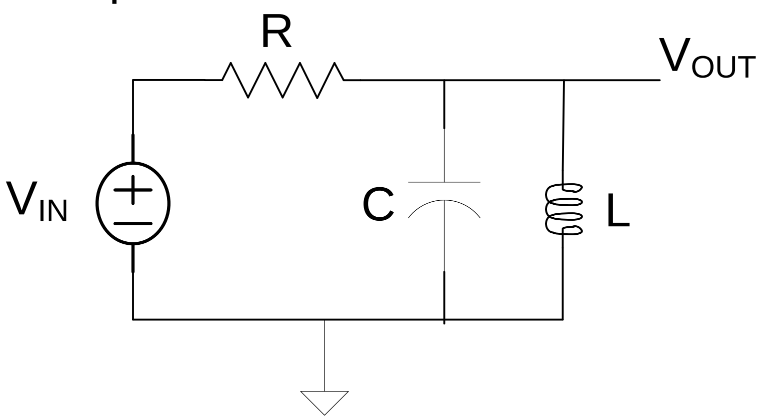
3 degrees of freedom

2 degrees of freedom for determining dimensionless transfer function

(impedance values scale)

$\omega_0 = ?$ Q = ? BW = ?

Example 1:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{RC}\frac{s}{s^2+s\left(\frac{1}{RC}\right)+\frac{1}{LC}}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \qquad Q = R\sqrt{\frac{C}{L}} \qquad BW = \frac{1}{RC}$$

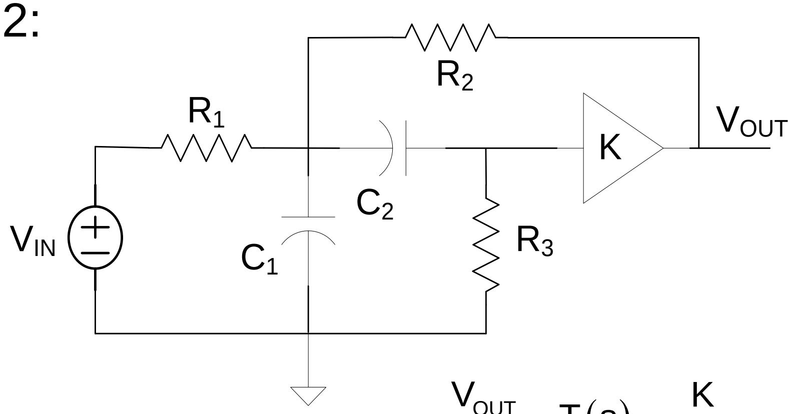
Can realize an arbitrary stable 2^{nd} order bandpass function within a gain factor

Simple design process (sequential but not independent control of ω_0 and Q)

If trimming is necessary, prefer to trim with a single resistor

Can’t trim this filter with single resistor

Example 2:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{R_1C_1}\frac{s}{s^2+s\left(\frac{1}{R_1C_1}+\frac{1}{R_3C_1}+\frac{1}{R_3C_2}+\frac{1\text{-}K}{R_2C_1}\right)+\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

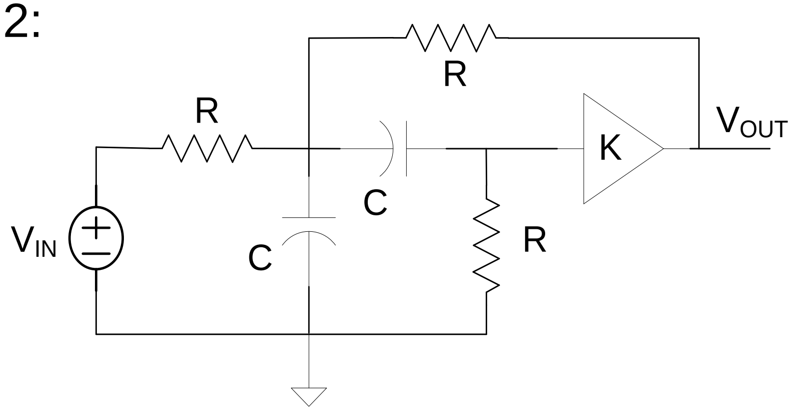
Second-order Bandpass Filter

6 degrees of freedom (effectively 5 because dimensionless)

Denote as a +KRC filter

$\omega_0 = ?$ Q = ? BW = ?

Example 2:



Equal R, Equal C Realization

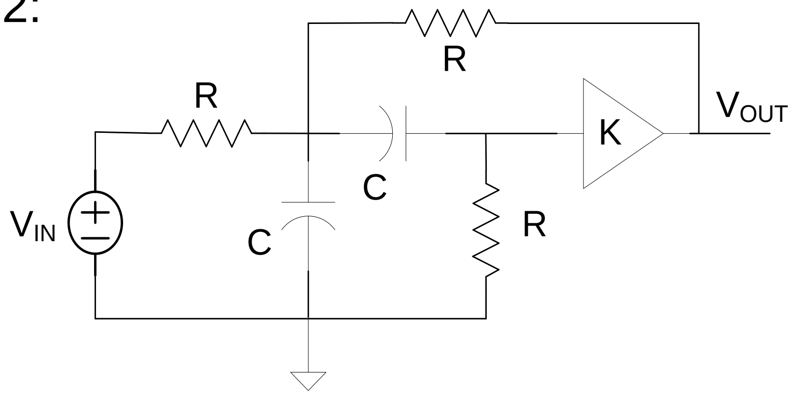
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{RC}\frac{s}{s^2 + s\left(\frac{4-K}{RC}\right) + \frac{2}{(RC)^2}}$$

$\omega_0 = ?$

Q = ?

BW = ?

Example 2:



Equal R, Equal C Realization

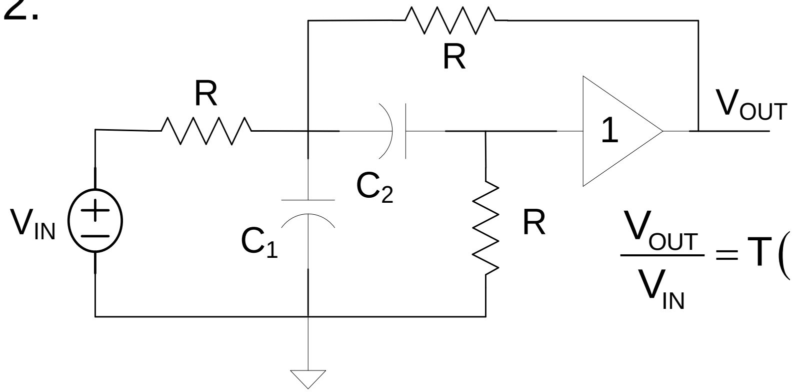
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{RC} \frac{s}{s^2 + s\left(\frac{4-K}{RC}\right) + \frac{2}{(RC)^2}}$$

$$\omega_0 = \frac{\sqrt{2}}{RC} \qquad Q = \frac{\sqrt{2}}{4-K} \qquad BW = \frac{4-K}{RC}$$

3 degrees of freedom (effectively 2 since dimensionless)

- Can satisfy arbitrary 2nd=order BP constraints within a gain factor with this circuit
- Very simple circuit structure
- Independent control of ω_0 and Q but requires tuning more than one component
- Can actually move poles in RHP by making K >4

Example 2:



Unity Gain, Equal R

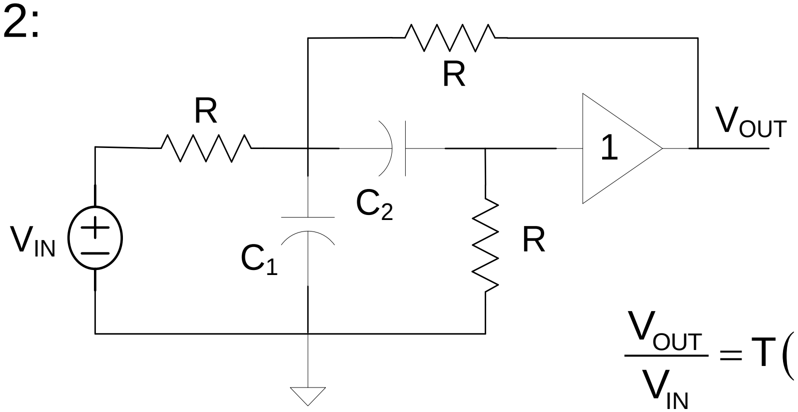
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{R\,C_1}\,\frac{s}{s^2 + s\left(\left[\frac{1}{R}\right]\left(\frac{2}{C_1} + \frac{1}{C_2}\right)\right) + \frac{2}{R^2 C_1 C_2}}$$

$\omega_0 = ?$

Q = ?

BW = ?

Example 2:



Unity Gain, Equal R

$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{R\,C_1}\frac{s}{s^2 + s\left(\left[\frac{1}{R}\right]\left(\frac{2}{C_1}+\frac{1}{C_2}\right)\right) + \frac{2}{R^2 C_1 C_2}}$$

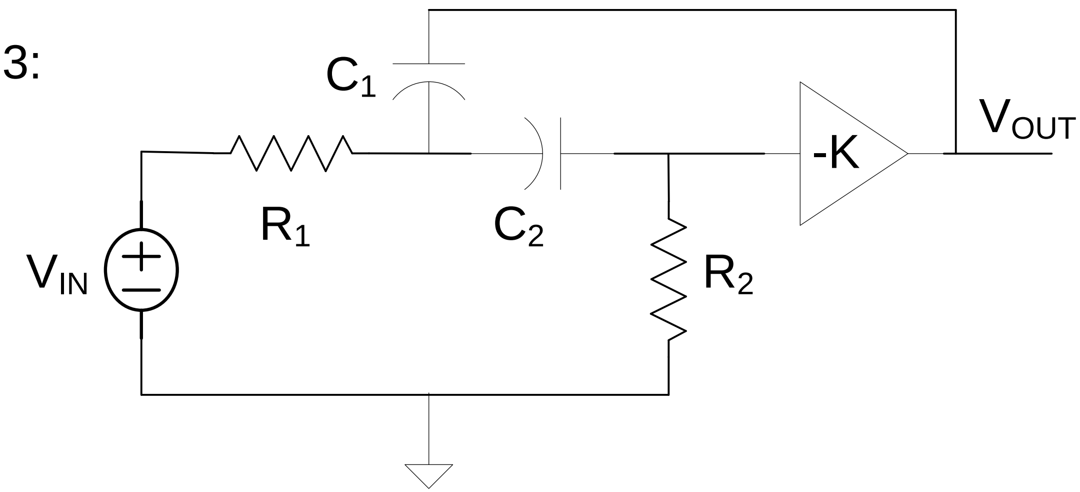
$$\omega_0 = \frac{\sqrt{2}}{R\sqrt{C_1 C_2}}$$

$$Q = \sqrt{2}\sqrt{\frac{C_2}{C_1}} + \frac{1}{\sqrt{2}}\sqrt{\frac{C_1}{C_2}}$$

$$BW = \left[\frac{1}{R}\right]\left(\frac{2}{C_1}+\frac{1}{C_2}\right)$$

Can't trim this filter with resistor

Example 3:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{K}{(1+K)R_1C_1}\frac{s}{s^2+s\left(\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right]\frac{1}{(1+K)}\right)+\frac{1}{(1+K)R_1R_2C_1C_2}}$$

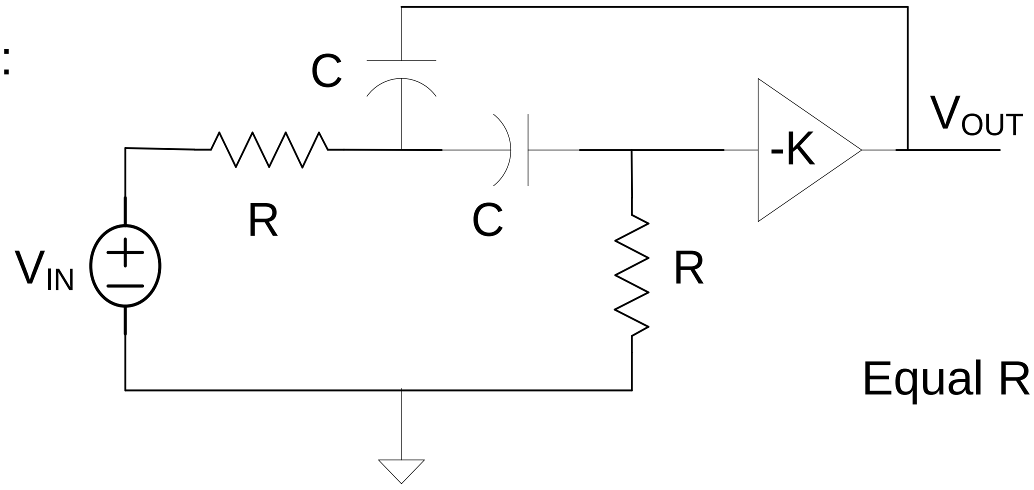
Second-order Bandpass Filter

5 degrees of freedom (4 effective since dimensionless)

Denote as a -KRC filter

$\omega_0 = ?$ Q = ? BW = ?

Example 3:



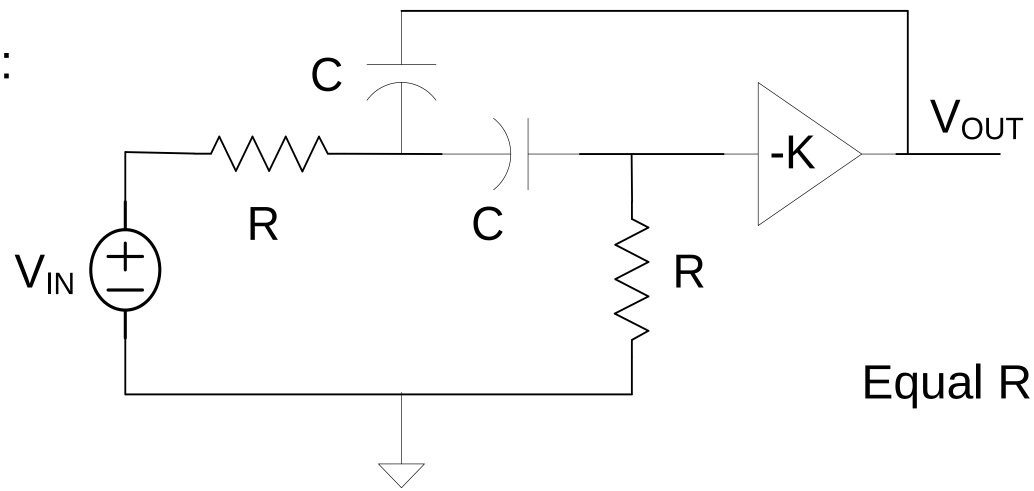
Equal R, Equal C Realization

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{K}{(1+K)RC}\frac{s}{s^2+s\left(\left[\frac{3}{RC}\right]\frac{1}{(1+K)}\right) + \frac{1}{(1+K)(RC)^2}}$$

3 degrees of freedom

$\omega_0 = ?$ Q = ? BW = ?

Example 3:



Equal R, Equal C Realization

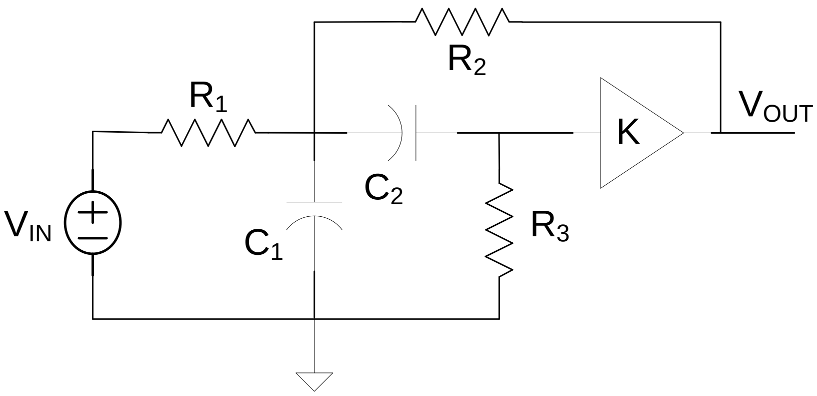
$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{K}{(1+K)RC}\frac{s}{s^2+s\left(\left[\frac{3}{RC}\right]\frac{1}{(1+K)}\right)+\frac{1}{(1+K)(RC)^2}}$$

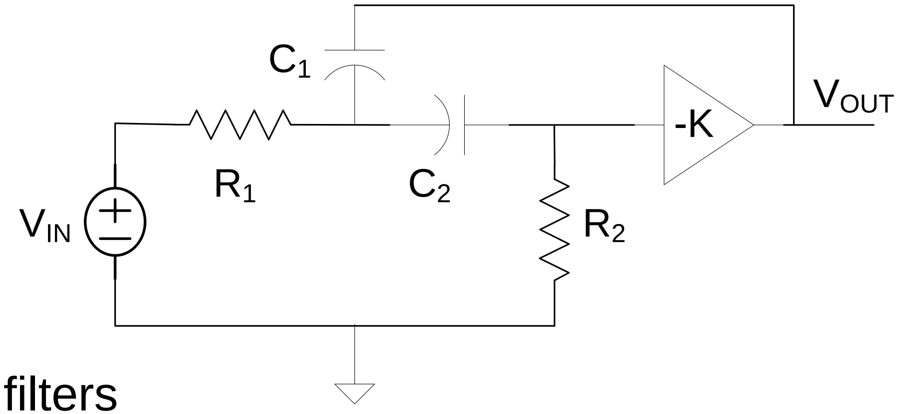
$$\omega_0 = \frac{1}{RC\sqrt{1+K}} \qquad Q = \frac{\sqrt{1+K}}{3} \qquad BW = \frac{3}{RC(1+K)}$$

3 degrees of freedom (2 effective since dimensionless)

- Can satisfy arbitrary 2nd=order BP constraints within a gain factor with this circuit
- Very simple circuit structure
- Simple design process (sequential but not independent control of ω_0 and Q, requires tuning of more than 1 component if Rs used)

Observation:





These are often termed Sallen and Key filters

Sallen and Key introduced a host of filter structures

Sallen and Key structures comprised of summers,
RC network, and finite gain amplifiers

These filters were really ahead of their time and appeared long before practical implementations were available

IRE TRANSACTIONS—CIRCUIT THEORY *March* 1955

A Practical Method of Designing RC Active Filters*

R. P. SALLEN† AND E. L. KEY†

Stay Safe and Stay Healthy !

End of Lecture 16